II. Solution by DR. O. E. GLENN, Drury College.

The theorem covering the general problem is due to Gauss,* and is the following: If p is any prime less than or equal to m, then the highest power of p dividing $m!$ is

$$p^{[m/p]+[m/p^2]+[m/p^3]+\dots\dots}=p^{\sum_{i=1}^{\infty}[m/p^i]}$$

where $[s/t]$ stands for the greatest integer in s/t. Applying this we have $2^{500+250+125+62+31+15+7+3+1}=2^{994}$, and similarly for the others.

Also solved by A. H. Holmes, and G. B. M. Zerr.

GEOMETRY.

263. Proposed by FREDERICK R. HONEY, Trinity College, Hartford, Conn.

Construct a sphere whose surface shall intersect the surface of any four given spheres in great circles.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Let P be the center of a circle intersecting two circles, centers C, C', in the extremities of diameters AB, $A'B'$, respectively. Draw through P a perpendicular to CC', intersecting it in D. Then, if r be the radius of the intersecting circle, we have

$$r^2=PC^2+CA^2=PC'^2+C'A'^2.$$

$$\therefore PD^2+DC^2+CA^2=PD^2+DC'^2+C'A'^2, \text{ and } DC^2-DC'^2=C'A'^2-CA^2.$$

Hence D is a fixed point, and the locus of P is consequently a fixed line. By rotating the figure about CC' we find that the locus of the center of a sphere intersecting two given spheres in great circles is a certain plane.

Constructing these planes for three pairs of the given spheres, each sphere being involved, we get a common point as the center of the required sphere, assuming that the centers of the given spheres are not coplanar.

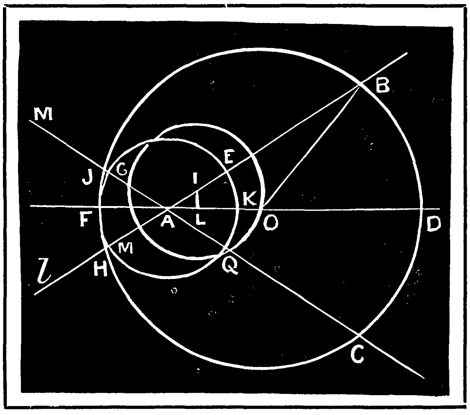
264. Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.

Let l and m be two straight lines intersecting in A. With A as center and any radius r describe a circle intersecting l and m in E, M and G, Q, respectively; and the bisector of the opposite angles formed by l and m in F and K. With I, the middle point of EA, as center, and radius, r, describe an arc intersecting the bisector of the opposite angles formed by l and m in O. With O as center, and radius $OA+r$ describe circle $FHCDBJF$; F and D the points of intersection of this circle with the bisector of opposite angle; H, B the intersections on l, and J, C on m. What is the ratio of arc HFJ to arc BD?

Disquisitiones Arithmeticae.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let 2θ be the angle formed by l, m. Also let A be the origin. Then $(\frac{1}{2}r\cos\theta, \frac{1}{2}r\sin\theta)$ are the coördinates of I where FD is the axis of abscissas.



$\therefore x^2+y^2-rx\cos\theta-ry\sin\theta=\frac{3}{4}r^2$ is the equation to the circle, center I.

$\therefore y^2-ry\sin\theta=\frac{3}{4}r^2$ or

$y=\frac{1}{2}r\sin\theta\pm\frac{1}{2}\sqrt{(3r^2+r^2\sin^2\theta)}$.

$\therefore OD=OA+r=y+r=r[1+\frac{1}{2}\sin\theta+\frac{1}{2}\sqrt{(3+\sin^2\theta)}]$.

Then if $\angle BOD=\phi$,

$$AO:BO=\sin(\phi-\theta):\sin\theta.$$

$$\therefore \sin(\phi-\theta)=\frac{[\sin\theta+\sqrt{(3+\sin^2\theta)}]\sin\theta}{2+\sin\theta+\sqrt{(3+\sin^2\theta)}}$$

This gives ϕ. Now $\text{arc}HFJ:\text{arc}BD=2\theta r:\phi DO$.

$\therefore \text{arc}HFJ:\text{arc}BD=2\theta:[2+\sin\theta+\sqrt{(3+\sin^2\theta)}]\phi$.

265. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

Find the Cartesian equation of a curve in a vertical plane such that a particle, sliding down the curve under the force of gravity alone, will require to pass from any point of beginning to the lowest point of the curve, a time proportional to the square of the distance to be traversed along the curve.

Solution by PROFESSOR WILLIAM HOOVER, Ph. D., Athens, Ohio.

With the usual notation, the equation of motion is

$$\frac{ds^2}{dt^2}=2gy \text{.........} (1).$$

Let $t=ks^2$, or $s\sqrt{k}=\sqrt{t}$ ....(2). This gives

$$\frac{d^2s}{dt^2}=\frac{1}{4kt}=\frac{1}{4k^2s^2}=2gy \text{.........}(3)\text{, or, } s=\frac{1}{2k\sqrt{2g}\sqrt{y}} \text{.........} (4).$$

Differentiating, $ds=-\dfrac{1}{4k\sqrt{2g}}\cdot\dfrac{1}{y^{\frac{3}{2}}}$ ....(5). But $dx^2+dy^2=ds^2$ ....(6).

Substituting (5) in (6), and arranging,

$$dx=\frac{\sqrt{(1-16k^2gy^3)}}{4k\sqrt{g}\,y^{\frac{3}{2}}}dy,$$

the differential equation of the curve, cartesian coördinates.

Also solved by G. B. M. Zerr. Professor Greenwood derives the *intrinsic* equation in the simple form $k^2=4s^3g\cos\phi$.